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Fluid impact on a solid boundary

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Abstract

The hydrodynamic impact due to a column of liquid hitting on a solid wall is analysed. The problem is solved using the complex velocity potential together with the boundary element method, based on the assumption that the fluid is inviscid and incompressible and the flow is irrotational. A stretched coordinate system is used, which is defined through the ratio of the normal Cartesian system to an appropriately chosen length scale varying with time. Numerical simulations are made for incoming liquid columns of various shapes. The results from the triangular column are compared with those obtained from a similarity solution and excellent agreement is found. It is shown that for a symmetrical liquid column with a shape defined by $f(x) = a^{1-\alpha}x^{\alpha}$, with $0<\alpha<1$ and $a>0$, the flow field and pressure distribution in the fluid after impact depend only on the parameter Ut/a: they do not depend separately on the speed U of the liquid column, the time t or the coefficient a.

Keywords: Liquid column; Impact; Stretched coordinate system

1. Introduction

As pointed out in previous papers (Wu et al., 2004; Wu, 2006), the fluid/structure impact problem has a unique feature. The collision usually starts from a single contact point if both the body and the liquid surface are smooth. At the initial stage of the impact, the contact zone between the liquid and the structure is small. The significant effect of the impact on the fluid flow will be confined to a small region near the contact point. However, all the physical parameters within the region, including velocity and pressure, can change rapidly. It is not easy to capture this change using numerical simulation based on the normal Cartesian system, unless extremely small elements are used. It is therefore argued (Wu et al., 2004; Wu, 2006) that an ideal approach is to use a stretched coordinate system. Indeed, for the water entry problem of wedges, Wu et al. (2004) and Wu (2006) defined a new coordinate system (ξ,η) based on the ratio of the Cartesian system (x,y) to the distance s that the body has travelled into water from the still level. This leads to the size of the elements used and the computational domain remaining more or less the same in the stretched system, while both of them vary with time in the physical system. This approach is clearly more rational and is one of the main reasons for the success of these analyses.

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Most previous papers on impact are for bodies moving towards a liquid with a flat surface. In particular, the body considered is of wedge shape [e.g., Dobrovol'skaya (1969), Greenhow (1982), Zhao and Faltinsen (1993)]. Korobkin (1996) considered a problem of an axisymmetrical liquid column hitting a plate. Although the compressibility of the liquid was included, the deformation of the liquid column during the impact was ignored, as the analysis focused on the 'acoustic stage'. Another example was the case considered by Smith et al. (2003) for liquid columns moving towards each other, but the focus was on the air cushion and on the period until 'touchdown'. A further example was the case of a droplet impact on a thin fluid layer during the initial stages (Howison et al., 2005). Here, we consider a column of two-dimensional liquid moving with speed U opposite to the direction of the x axis towards a stationary solid flat surface, or a plate moving towards the initial stationary liquid (see Fig. 1). Without loss of generality, we may consider a symmetric case and the upper part the liquid surfaces is described by $y = f(x) \geqslant 0$. It is obvious that an ideal approach is to use the stretched system. It is important to realize, however, when $f(x)$ is not a linear function, that $\eta = f(s\xi)/s \neq f(\xi)$. Thus, during the impact, the free surface of the liquid column will change in the stretched system for two reasons. One is due to deformation caused by the impact, which is physical, and the other is caused by the variation of s, which is purely mathematical. Both changes need to be taken into account in the simulation.

The choice of s is not unique. Wu et al. (2004) and Wu (2006) used the distance travelled into the water by the body. This worked quite well for the water entry problem of wedges. Here, such a choice could be problematic in the numerical simulation. It would be more illustrative to assume that the liquid column is not moving and its tip is at $x = 0$. The body, i.e., the wall, is moving with U in the x direction and it touches the tip of the liquid at $t = 0$. When the deformation caused by the fluid motion is ignored, the intersection point of the free surface and body surface will be at $\xi = 1$ and $\eta = f(s)/s$ with $s = Ut$. At the beginning of the simulation, s is very small and the coordinates of the intersection point can be approximated as $[1, f'(0)]$. On the other hand, when the shape of the water column is a curve instead of a straight line, $f'(0)$ can be infinite. This means that the wetted surface of the body in the stretched system is infinitely large, which is impractical for numerical simulation.

To resolve this problem, one of the options is to choose $s = f(Ut)$. The intersection point then becomes $[Ut/f(Ut), 1]$ when the motion of the liquid is ignored. As a result, when t is small, the intersection point can be approximated as $[1/f'(0), 1]$. This gives a finite wetted surface of the body and is much more suitable for the numerical simulation. One of the disadvantages of this choice of s is that the location of the body is not fixed and it changes in terms of $Ut/f(Ut)$, but this can be overcome by redefining ξ as $\xi - Ut/f(Ut)$. Real difficulty may arise if $f(x) \approx 0$ at some point with $x \gg 0$, which can happen when $f(x)$ is not a monotonic function. A more appropriate choice for s in this case would be based on the area covered by $f(x)$ between $(0, Ut)$, or $s = \int_0^{Ut} f(x)\,dx/(Ut)$. When t is small, $s \approx f(Ut)$. This is similar to but not the same as when $s = f(Ut)$. Obviously, when t increases, further difference between these two will develop.

In the following sections, the complex velocity potential based on a boundary element approach is first outlined. The method is then used to solve in the time domain the problem of triangular liquid columns with different angles. The results are compared with those obtained from the similarity solutions and excellent agreement is found. The method is then used to simulate impact of liquid columns with $f(x) = a^{1-\alpha} x^{\alpha}$, $0 < \alpha < 1$. It is shown that the result after impact does not depend on U, t and a separately in the stretched coordinate system; it only depends on the variable Ut/a. This is consistent with the PI theorem.

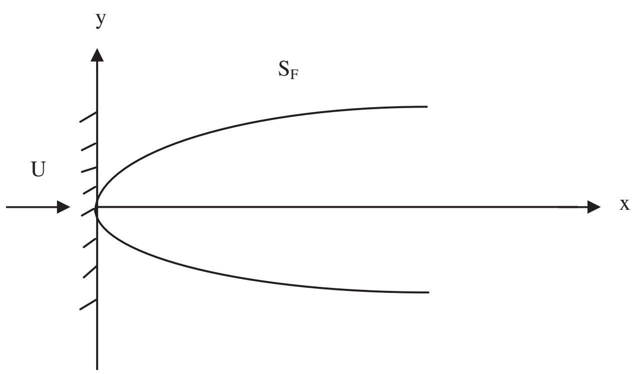


Fig. 1. Sketch of the liquid column impact problem.

2. Governing equations and numerical procedure

We consider the hydrodynamic problem of fluid impact on a solid wall with speed U. It is dynamically equivalent to the case when the column of liquid is stationary and the wall is moving with the same speed in the opposite direction, as shown in Fig. 1. A Cartesian coordinate system $O-xy$ is defined in which the origin of the system is fixed at the tip of the column when the body touches the liquid. The fluid is assumed to be incompressible and inviscid, and the flow is assumed to be irrotational. A velocity potential ϕ can then be introduced. From the continuity equation, ϕ satisfies

$$\nabla^2\phi = 0 \tag{1}$$

in the fluid domain R. On the body surface S_0 or $x = Ut$, we have

$$\frac{\partial\phi}{\partial n} = Un_x, \tag{2}$$

where $n = (n_x,n_y)$ is the normal vector of the body surface pointing out of the fluid domain, which is in fact $(-1,0)$ for the case of a wall along the y direction. The kinematic and dynamic conditions on the free surface S_F or $y = \zeta$ can be written as

$$\frac{\partial\zeta}{\partial t} = \phi_y - \phi_x\zeta_x, \tag{3}$$

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}\nabla\phi\nabla\phi = 0. \tag{4}$$

In the Lagrangian framework, these become

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\partial\phi}{\partial x}, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\partial\phi}{\partial y}, \tag{5}$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{1}{2}\nabla\phi\nabla\phi. \tag{6}$$

Without loss of generality, we can assume that the liquid column is symmetric about $y = 0$. Thus, we have $\partial\phi/\partial y = 0$ on $y = 0$. For the initial condition, we assume that, at the moment of contact, the free surface is described by $y = f(x)$ and $\phi[x,f(x)] = 0$.

As discussed in the Section 1, it is appropriate to use a stretched coordinate system for this kind of problem. We may define

$$\phi(x, y, t) = Us\varphi(\xi, \eta, t), \tag{7}$$

where $\xi = x/s$ and $\eta = y/s$. Eq. (1) obviously has the same form in the stretched system, while Eqs. (2), (5) and (6) become

$$\frac{\partial\varphi}{\partial n} = n_\xi, \tag{8}$$

$$\frac{\mathrm{d}(s\xi)}{\mathrm{d}t} = U\frac{\partial\varphi}{\partial\xi}, \quad \frac{\mathrm{d}(s\eta)}{\mathrm{d}t} = U\frac{\partial\varphi}{\partial\eta}, \tag{9}$$

$$\frac{\mathrm{d}(s\varphi)}{\mathrm{d}t} = \frac{U}{2}(\varphi_\xi^2 + \varphi_\eta^2). \tag{10}$$

As the problem is two dimensional, it is convenient to adopt the complex potential $w = \varphi + \mathrm{i}\psi$, where ψ is the stream function. The method has been previously used in many free-surface-related problems (Lin et al., 1985; Greenhow, 1982; Wu and Eatock Taylor, 1995, 2003; Wu, et al., 2004; Wu, 2006). It is based on Cauchy's theorem which gives

$$\oint \frac{w}{z - z_0}\,\mathrm{d}z = 0, \tag{11}$$

where $z = x + \mathrm{i}y$ and z_0 is a point outside of the fluid domain R. The integration in Eq. (11) is along the fluid boundary. On the fluid boundary, we divide the surface into segments with n nodes. The complex function can be written in terms of the interpolation function

$$w = \sum_{j=1}^{n} w_j N_j(z), \tag{12}$$

where w_j are the complex potentials at the nodes. When linear interpolation is used, we have

$$N_j(z) = \begin{cases} (z - z_{j+1})/(z_j - z_{j+1}), & z \in (z_j, z_{j+1}), \\ (z - z_{j-1})/(z_j - z_{j-1}), & z \in (z_{j-1}, z_j), \\ 0, & z \notin (z_{j-1}, z_{j+1}). \end{cases} \tag{13}$$

Substituting Eqs. (12) and (13) into (11), letting z_0 approach node z_k and using the boundary conditions, we have

$$\sum_{j=1}^{n} A_{kj}\phi_j|_{j\in S_0+S_C} + \mathrm{i}\sum_{j=1}^{n} A_{kj}\psi_j|_{j\in S_F} = -\sum_{j=1}^{n} A_{kj}\phi_j|_{j\in S_F} - \mathrm{i}\sum_{j=1}^{n} A_{kj}\psi_j|_{j\in S_0+S_C}, \tag{14}$$

where

$$A_{kj} = \frac{z_k - z_{j-1}}{z_j - z_{j-1}}\ln\left(\frac{z_j - z_k}{z_{j-1} - z_k}\right) + \frac{z_k - z_{j+1}}{z_j - z_{j+1}}\ln\left(\frac{z_{j+1} - z_k}{z_j - z_k}\right). \tag{15}$$

In Eq. (14), the terms on the right-hand side are known from the boundary conditions while the terms on the left are unknown. At the intersection in particular, both the stream function and the potential are known, and they are both moved to the right-hand side of the equation. S_C in the equation is a control surface far away from the wall.

When the solution of Eq. (14) has been found, the pressure can in theory be obtained from the Bernoulli equation

$$p = -\rho\phi_t - \frac{1}{2}\rho(\phi_x^2 + \phi_y^2). \tag{16}$$

The difficulty is that ϕ_t is in fact still not known directly. It is, nevertheless, another harmonic function, as it satisfies the Laplace equation. On the free surface, $p = 0$ gives

$$\phi_t = -\tfrac{1}{2}(\phi_x^2 + \phi_y^2). \tag{17}$$

On the body surface, we have (Wu, 1998)

$$\frac{\partial\phi_t}{\partial n} = -U\frac{\partial\phi_x}{\partial n}. \tag{18}$$

To solve the boundary value problem for ϕ_t the stretched system can also be used. In fact, we can write $\phi_t = U^2\chi(\xi,\eta,t)$. Eqs. (17) and (18) then become

$$\chi = -\tfrac{1}{2}(\varphi_\xi^2 + \varphi_\eta^2) \tag{19}$$

and

$$\frac{\partial\chi}{\partial n} = -\frac{\partial\varphi_\xi}{\partial n}, \tag{20}$$

respectively. Once χ is found, the force F on the wall can be found from

$$F = -\rho\int_{S_0}\left(\phi_t + \frac{1}{2}\nabla\phi\nabla\phi\right)n_x\,\mathrm{d}S|_{(x,y)} = \rho U^2 s\int_{S_0}\left(\chi + \frac{1}{2}\nabla\varphi\nabla\varphi\right)n_\xi\,\mathrm{d}S|_{(\xi,\eta)}. \tag{21}$$

3. Similarity solution

In general, φ defined in the stretched coordinate system is still a function of time. There are cases, however, in which the function is time-independent because the solution is self-similar. One of these cases is when the liquid column is of triangular shape (Dobrovol'skaya, 1969). We take $s = Ut$. Because $\partial\varphi/\partial t = 0$ in the stretched coordinate system, it is obvious that

$$\phi_t = U^2(\varphi - \xi\varphi_\xi - \eta\varphi_\eta). \tag{22}$$

Thus in the Eulerian form, the free-surface boundary conditions in Eqs. (3) and (4) can be written as

$$\eta - \xi\eta_\xi = \varphi_\eta - \varphi_\xi\eta_\xi, \quad \varphi - \xi\varphi_\xi - \eta\varphi_\eta + \frac{1}{2}(\varphi_\xi^2 + \varphi_\eta^2) = 0. \tag{23,24}$$

As in Wu et al. (2004), for numerical solution, these two equations can be converted into two integral equations:

$$\eta = \xi\left(\int_{\xi_0}^{\xi} \frac{\varphi_\xi \eta_\xi - \varphi_\eta}{\xi^2}\,\mathrm{d}\xi + \frac{\eta_0}{\xi_0}\right), \qquad \varphi = \xi\left(\int_{\xi_0}^{\xi} \frac{\eta_\xi \varphi_\xi \varphi_\eta + 0.5(\varphi_\xi^2 - \varphi_\eta^2)}{\xi^2}\,\mathrm{d}\xi\right), \tag{25,26}$$

where (ξ_0,η_0) is a point chosen sufficiently far away from the body that the fluid there is assumed to be undisturbed.

If a jet is developed along the body, the above method is matched with the shallow water solution for the jet. The shallow water solution is based on the assumption that the jet is sufficiently thin and only linear terms are needed in the Taylor expansion of φ and η in terms of ξ. This leads to Wu et al. (2004)

$$\eta = \frac{\eta_e - \eta_s}{\xi_e - \xi_s}(\xi - \xi_s) + \eta_s, \qquad \phi = \xi - \xi_s + \eta_e(\eta - \eta_s) + \phi_s, \tag{27,28}$$

where (ξ_s,η_s) is the starting point of the jet and (ξ_e,η_e) is the intersection point between the body and the jet, and $\varphi_s = \varphi(\xi_s,\eta_s)$. The point (ξ_s,η_s) can be chosen based on the distance to the body or the angle between the body surface and the free surface (Lu et al., 2000). For the intersection point, $\xi_e = 1$ as it is on the body surface, and

$$\eta_e = \eta_s + \sqrt{\eta_s^2 - 2\varphi_s + 2\xi_s - 1} \tag{29}$$

as can be obtained from the dynamic free-surface boundary condition. Using Eqs. (27) and (28), we can also see that

$$\varphi_e = \varphi(1,\eta_e) = \frac{1}{2}(\eta_e^2 + 1), \tag{30}$$

which links the potential at the intersection with its coordinates.

4. Numerical results

We first consider the case of a triangular liquid column and choose $s = Ut$. Because the flow is self similar, it allows the problem to be solved using both the time-domain method in Section 2 and the similarity solution procedure in Section 3. The results can be used for validation. In the time domain method, the solution starts from a very small t_s. The free surface remains unchanged at this instant, or $\eta = \tan(\beta)\xi$, where 2β is the angle of the liquid column. The initial potential on the free surface is zero, or $\varphi[\xi,\tan(\beta)\xi,t_s] = 0$. Strictly speaking, this assumption is not valid unless $t_s = 0$. This is because, no matter how small t_s is, the initial solution should be self-similar. However, as the disturbance to the liquid is confined to a tiny region in the physical domain, the result is not expected to have a significant effect at a later stage when $t = T \gg t_s$, or $T/t_s \gg 1$. In numerical simulation, this intuitive hypothesis can be verified if the result at $t = T$ is not significantly affected when even smaller t_s is used.

To take the speed out of the simulation, we can rewrite Eqs. (9) and (10) as

$$\frac{\mathrm{d}(s\xi)}{\mathrm{d}s} = \frac{\partial\varphi}{\partial\xi}, \quad \frac{\mathrm{d}(s\eta)}{\mathrm{d}s} = \frac{\partial\varphi}{\partial\eta}, \tag{31}$$

$$\frac{\mathrm{d}(s\varphi)}{\mathrm{d}s} = \frac{1}{2}(\varphi_\xi^2 + \varphi_\eta^2). \tag{32}$$

The time stepping can now be achieved through s. The advantage of this is that the unit of s and indeed units of all other parameters have no direct effect on the simulation. We start the simulation at $s = 0.001$. Figs. 2–4 give results at $\beta = 80^\circ$, 60°, 45° for the shape of the liquid column and the pressure distribution nondimensionalized by ρU^2 on the wall after impact. The time-domain solution corresponds to $s = 1$. It can be seen that the results from the two methods are in good agreement. As expected, the major deformation of the liquid surface is confined near the wall and a jet can be developed along the wall. The pressure increases along the wall from the centre. It reaches a peak near the jet, then drops to the ambient pressure in the jet. The change of the pressure can be very rapid, especially when β is close to 90°, which is very similar to the pressure distribution over a wedge in the water entry problem.

We next consider the case of a parabolic liquid column, or $y = \sqrt{ax}$, with $a>0$, and choose $s = f(Ut) = \sqrt{aUt}$; a here has obviously the dimension of the length scale. In the stretched coordinate system, the shape of the liquid column becomes $\eta = \sqrt{(a/s)\xi}$. If we further define $r = 2s/a$ which is a nondimensional parameter, the shape of the liquid column becomes $\eta = \sqrt{2\xi/r}$ and the location of the moving wall is at $\xi = Ut/s = r/2$. Similar to Eqs. (31) and (32), we

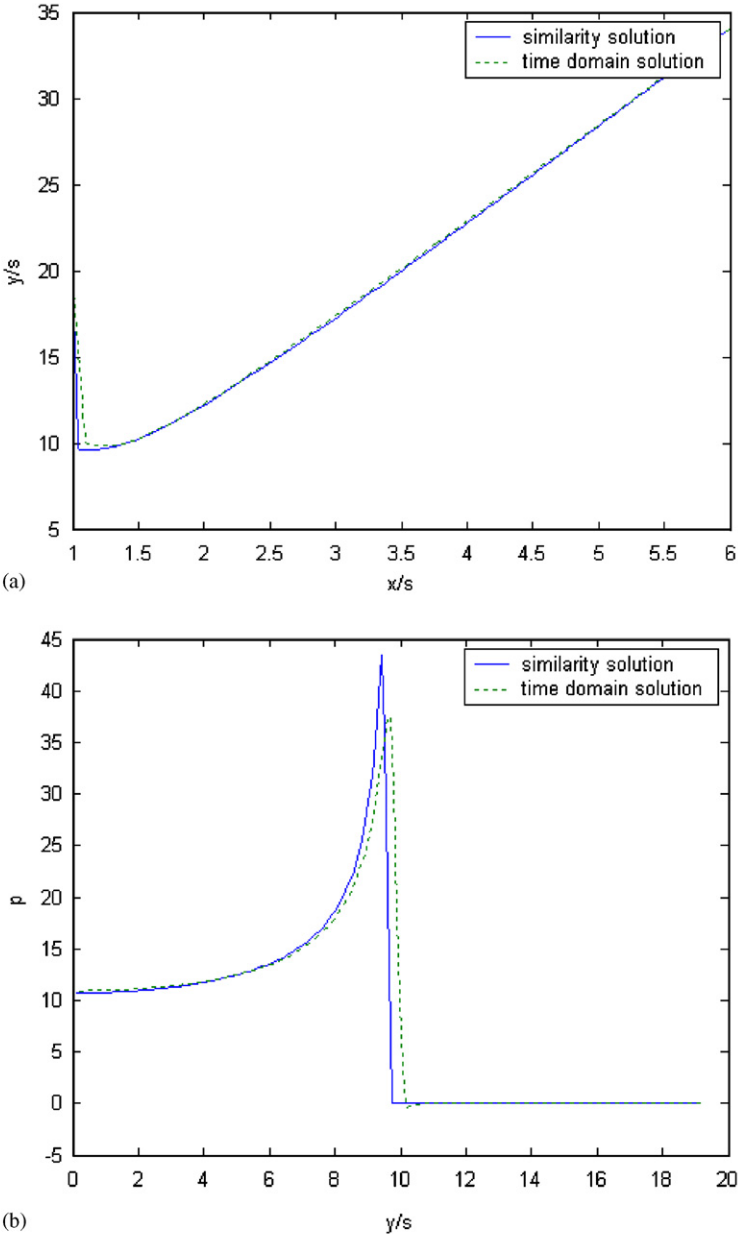


Fig. 2. (a) Liquid column surface profile ($\beta = 80^\circ$), (b) pressure distribution ($\beta = 80^\circ$).

can rewrite (9) and (10) as

$$\frac{\mathrm{d}(r\xi)}{\mathrm{d}r} = r\frac{\partial\varphi}{\partial\xi}, \quad \frac{\mathrm{d}(r\eta)}{\mathrm{d}r} = r\frac{\partial\varphi}{\partial\eta}, \tag{33}$$

$$\frac{\mathrm{d}(r\varphi)}{\mathrm{d}r} = \frac{r}{2}(\varphi_\xi^2 + \varphi_\eta^2), \tag{34}$$

which are fully nondimensional. All these equations show that a does not directly affect the results from the simulation. The results will be the same if r is the same, which means that the results depend on only $r = 2\sqrt{Ut/a}$, not on U, t and a individually, which can of course be expected from the PI theorem. It is, however, important to notice that the liquid shape $\eta = \sqrt{2\xi/r}$ changes with r, in addition to the change caused by the wall, as discussed in the introduction.

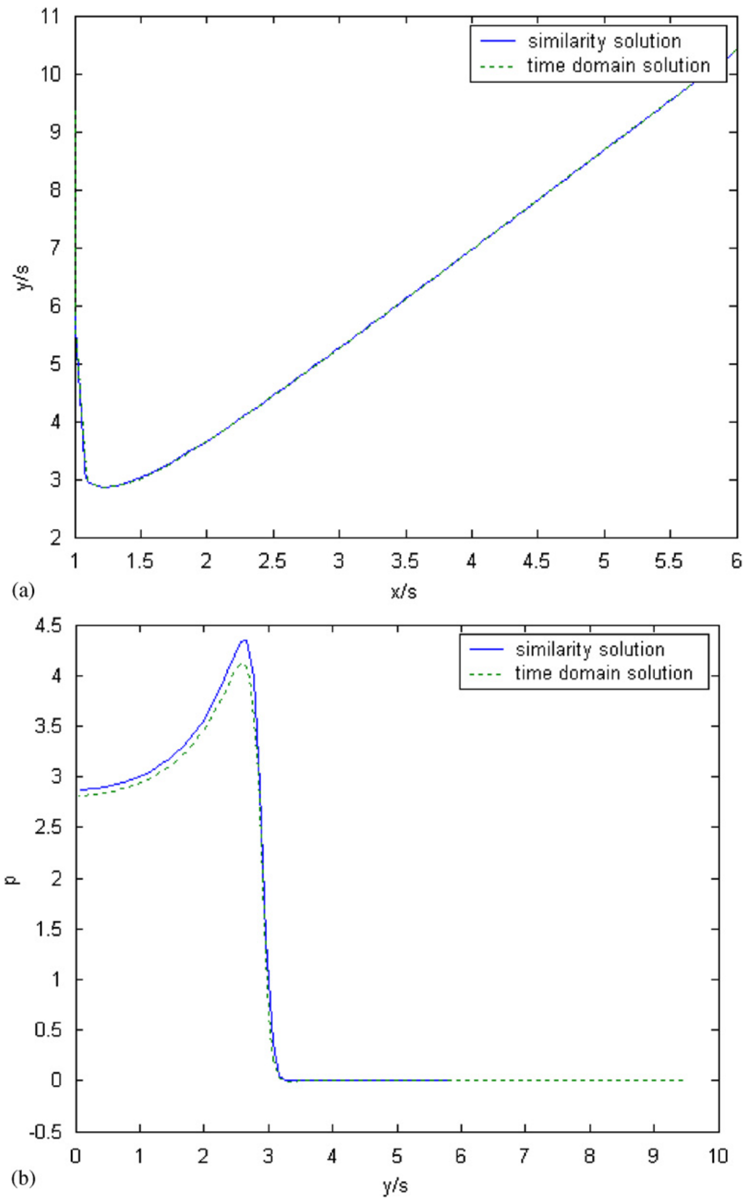


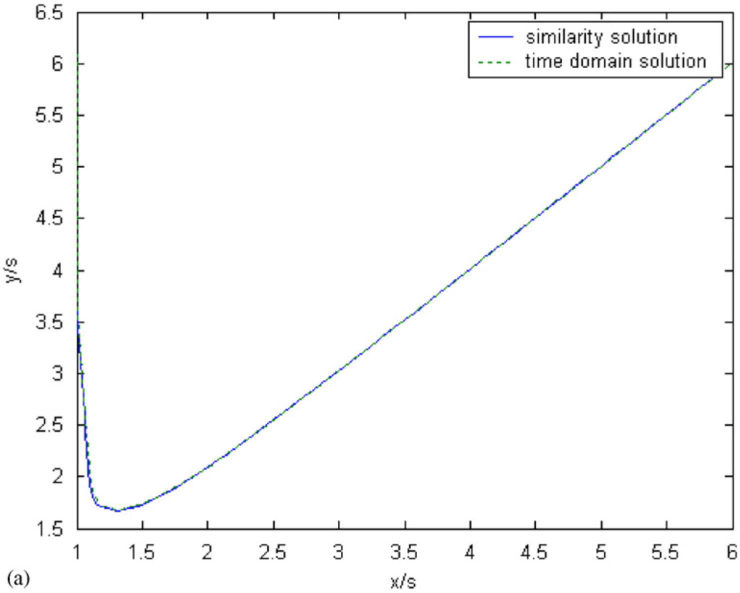
Fig. 3. (a)Liquid column surface profile ($\beta = 60^\circ$), (b) pressure distribution ($\beta = 60^\circ$).

Figs. 5(a) and (b) give the profile of the liquid surface and the pressure distribution on the wall at $r = 1.0, 1.5, 2.0$. The horizontal axis is measured from the rigid wall through redefining ξ as $\xi - Ut/s$. Unlike in the triangular case, these curves are not on the same line as the problem is no longer self-similar. The maximum pressure occurs at $y = 0$, which also differs from the cases of triangular liquid columns considered in Figs. 2–4. The maximum pressure decreases with r.

The above case may be generalized. If the surface of the water column is defined as $f(x) = a^{1-\alpha}x^{\alpha}$ with $0<\alpha<1$, we have $s = a^{1-\alpha}(Ut)^{\alpha}$. Letting $r = \alpha^{\alpha/(\alpha-1)}s/a = a^{\alpha/(1-\alpha)}(Ut/a)^{\alpha}$, we obtain the shape of the liquid column as $\eta = (\xi/\alpha)^{\alpha}/r^{1-\alpha}$ and the location of the wall at $\xi = \alpha r^{(1-\alpha)/\alpha}$. Correspondingly, Eqs. (9) and (10) become

$$\frac{\mathrm{d}(r\xi)}{\mathrm{d}r} = r^{(1-\alpha)/\alpha}\frac{\partial\varphi}{\partial\xi}, \quad \frac{\mathrm{d}(r\eta)}{\mathrm{d}r} = r^{(1-\alpha)/\alpha}\frac{\partial\varphi}{\partial\eta}, \tag{35}$$

$$\frac{\mathrm{d}(r\varphi)}{\mathrm{d}r} = \frac{1}{2}r^{(1-\alpha)/\alpha}(\varphi_{\xi}^2 + \varphi_{\eta}^2). \tag{36}$$



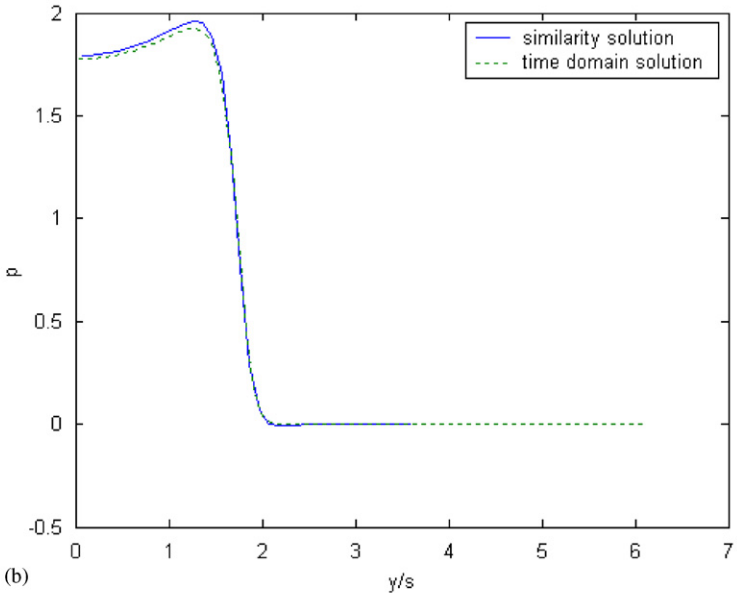
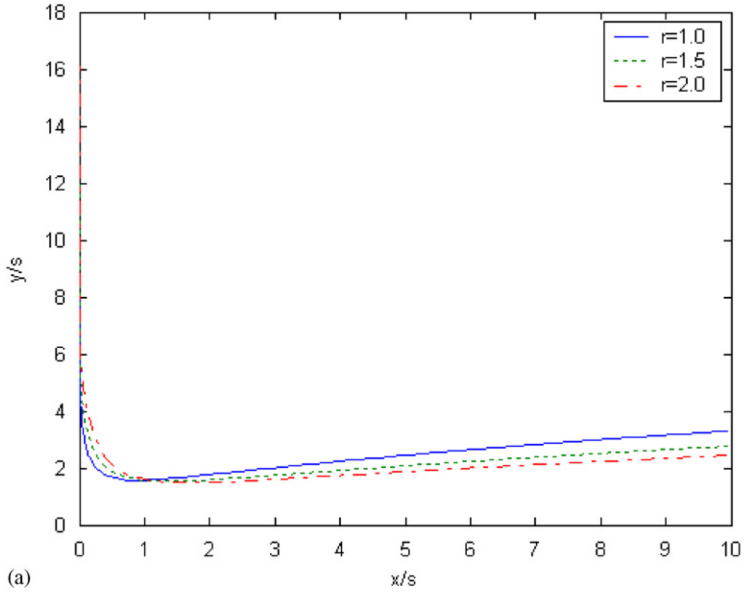


Fig. 4. (a) Liquid column surface profile ($\beta = 45°$), (b) pressure distribution ($\beta = 45°$).

These equations show that the result in the stretched system depends on r but not U, t and a individually. Figs. 6(a) and (b) give the profile and the pressure distribution at $r = 2$ with $\alpha = 1/2, 1/3, 1/4$. The maximum pressure decreases with α.

5. Conclusions

The impact problem of a two-dimensional liquid column on a rigid wall has been solved based on velocity potential theory. The success of the analysis is due to the adoption of the stretched coordinate system. The choice



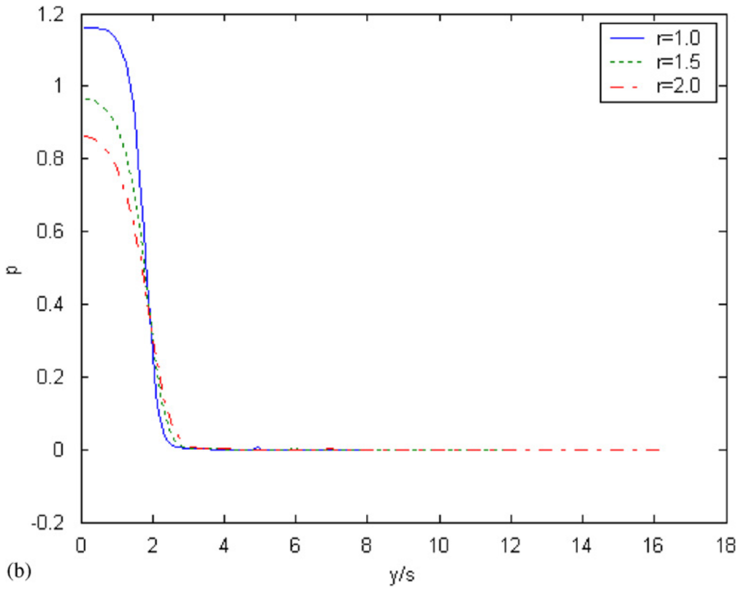
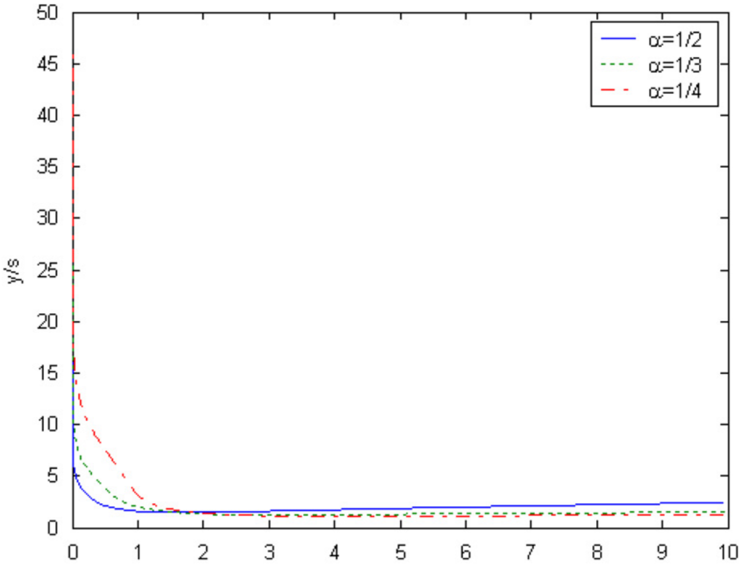


Fig. 5. (a) Liquid column surface profile with $y=\sqrt{ax}$, (b) pressure distribution with $y=\sqrt{ax}$.

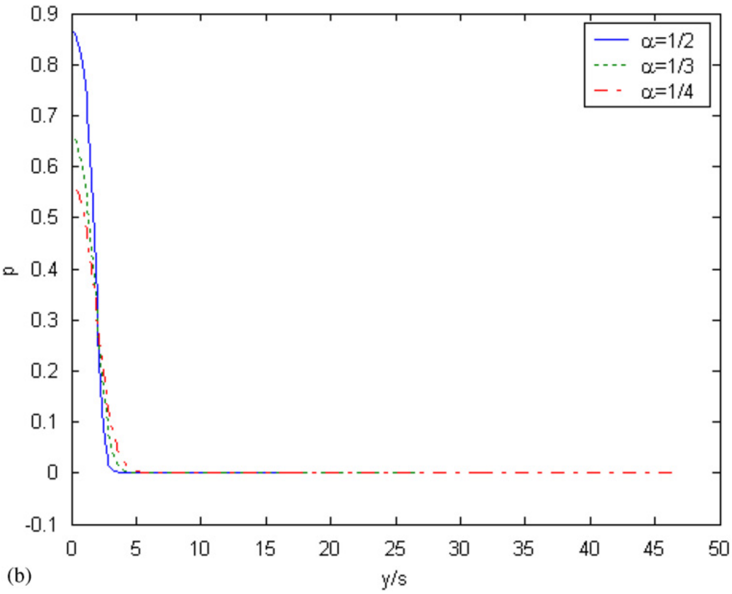
of s for scaling is particularly important. For a symmetric smooth liquid column with a surface profile given by $y=f(x)$, $s=f(Ut)$ has been found to be a suitable choice. This can automatically be extended to an asymmetrical case with

$$y=\begin{cases} f_1(x), & y>0, \\ -f_2(x), & y<0, \end{cases}$$

if s is chosen through the width of the liquid column, i.e., $s=f_1(x)+f_2(x)$. Indeed, this method can be further extended to many other problems including oblique impact if s is chosen in a similar way.



(a)



(b)

Fig. 6. (a) Liquid column surface profile at $r = 2$ with $y = a^{1-\alpha}x^{\alpha}$, (b) pressure distribution at $r = 2$ with $y = a^{1-\alpha}x^{\alpha}$.

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